Mathematische Hilfsmittel für Tedy

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$$\oint\!\!\!\oint_S \vec{F}d\vec{a} = \iiint_G \mathrm{div}\vec{F}dV \qquad (1.14)$$

$$\oint\!\!\!\oint_S Ud\vec{a} = \iiint_G \mathrm{grad}UdV \qquad (1.27)$$

$$\oint_K \vec{F}d\vec{r} = \iint_S \mathrm{rot}\vec{F}d\vec{a} \qquad (1.35a)$$

$$\iiint \mathrm{rot}\vec{F}dV = \oint\!\!\!\oint (\vec{n}\times\vec{F})da \qquad (1.41)$$

Quellfrei $\Leftrightarrow \mathrm{div}\vec{F} = 0$

Wirbelfrei $\Leftrightarrow \mathrm{rot}\vec{F} = \vec{0}$

	Kartesische Koordinaten	**Zylinderkoordinaten**	**Kugelkoordinaten**
	$(\vec{e}_x;\vec{e}_y;\vec{e}_z)$	$(\vec{e}_\rho;\vec{e}_\alpha;\vec{e}_z)$	$(\vec{e}_r;\vec{e}_\vartheta;\vec{e}_\alpha)$
$\mathrm{grad}\varphi =$	$\begin{pmatrix} \frac{d\varphi}{dx} \\ \frac{d\varphi}{dy} \\ \frac{d\varphi}{dz} \end{pmatrix}$	$\begin{pmatrix} \frac{d\varphi}{d\rho} \\ \frac{1}{\rho}\frac{d\varphi}{d\alpha} \\ \frac{d\varphi}{dz} \end{pmatrix}$	$\begin{pmatrix} \frac{d\varphi}{dr} \\ \frac{1}{r}\frac{d\varphi}{d\vartheta} \\ \frac{1}{r\sin\vartheta}\frac{d\varphi}{d\alpha} \end{pmatrix}$
$\mathrm{div}\vec{F} =$	$\frac{dF_x}{dx}+\frac{dF_y}{dy}+\frac{dF_z}{dz}$	$\frac{1}{\rho}\left(\frac{d(\rho F_\rho)}{d\rho}+\frac{dF_\alpha}{d\alpha}\right)+\frac{dF_z}{dz}$	$\frac{1}{r^2}\frac{d(r^2F_r)}{dr}+\frac{1}{r\sin\vartheta}\left(\frac{d(F_\vartheta\sin\vartheta)}{d\vartheta}+\frac{dF_\alpha}{d\alpha}\right)$
$\mathrm{rot}\vec{F} =$	$\begin{pmatrix} \frac{dF_z}{dy}-\frac{dF_y}{dz} \\ -\left(\frac{dF_z}{dx}-\frac{dF_x}{dz}\right) \\ \frac{dF_y}{dx}-\frac{dF_x}{dy} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\rho}\frac{dF_z}{d\alpha}-\frac{dF_\alpha}{dz} \\ -\left(\frac{dF_z}{d\rho}-\frac{dF_\rho}{dz}\right) \\ \frac{1}{\rho}\left(\frac{d(\rho F_\alpha)}{d\rho}-\frac{dF_\rho}{d\alpha}\right) \end{pmatrix}$	$\begin{pmatrix} \frac{1}{r\sin\vartheta}\left(\frac{d(F_\alpha\sin\vartheta)}{d\vartheta}-\frac{dF_\vartheta}{d\alpha}\right) \\ -\frac{1}{r}\left(\frac{d(rF_\alpha)}{dr}-\frac{1}{\sin\vartheta}\frac{dF_r}{d\alpha}\right) \\ \frac{1}{r}\left(\frac{d(rF_\alpha)}{dr}-\frac{dF_r}{d\vartheta}\right) \end{pmatrix}$
$d\vec{r} =$	$dx\vec{e}_x+dy\vec{e}_y+dz\vec{e}_z$	$d\rho\vec{e}_\rho+\rho d\alpha\vec{e}_\alpha+dz\vec{e}_z$	$dr\vec{e}_r+rd\vartheta\vec{e}_\vartheta+r\sin\vartheta d\alpha\vec{e}_\alpha$
$d\vec{a} =$	$dydz\vec{e}_x = dxdz\vec{e}_y = dxdy\vec{e}_z$	$\rho d\alpha dz\vec{e}_\rho = d\rho dz\vec{e}_\alpha = \rho d\rho d\alpha\vec{e}_z$	$r^2\sin\vartheta d\vartheta d\alpha\vec{e}_r = r\sin\vartheta drd\alpha\vec{e}_\vartheta = rdrd\vartheta\vec{e}_\alpha$
$dV =$	$dxdydz$	$\rho d\rho d\alpha dz$	$r^2\sin\vartheta drd\vartheta d\alpha$

Skalares Potential: $\mathrm{grad}\varphi = -\vec{F}$

Vektorpotential: $\mathrm{rot}\vec{A} = \vec{F}$

Feldliniengleichung: $\vec{F}\times d\vec{r} = \vec{0}$