# Elektrotechnik Formeln
## 1. und 2. Semester
von Gerald Meier

# 1 Elektrisches und magnetisches Feld

## 1.1 Elektrisches Feld

elektrische Feldstärke: $\vec{E} = \frac{\vec{F}}{q}$

elektrisches Feld einer Punktladung: $\vec{E} = \frac{q}{4\pi \cdot \varepsilon_0 \cdot r^2} \cdot \vec{e}_r$

$\varepsilon_0 = 0{,}885419 \cdot 10^{-11}\ \text{As}/\text{Vm}$

Kraft auf Ladung: $\vec{F} = q \cdot \vec{E}$

Spannung: $U_{12} = \int_{P1}^{P2} \vec{E} \cdot d\vec{r}$

Arbeit: $W = q \cdot U_{12}$

Ladung: $Q = \varepsilon_0 \cdot \oiint_A \vec{E} \cdot d\vec{a}$

Driftgeschwindigkeit der Elektronen: $\vec{v} = -b \cdot \vec{E}$

Raumladungsdichte: $\rho_V = \frac{\Delta Q_V}{\Delta V} = -e \cdot n$

Stromdichte: $\vec{J} = \rho_V \cdot \vec{v} = e \cdot n \cdot b \cdot \vec{E} := \kappa \cdot \vec{E}$

Stromstärke: $|i| = \frac{|\Delta Q|}{\Delta t}$

$i = \iint \vec{J} \cdot d\vec{a}$

daraus ohm’sches Gesetz: $u = \frac{l}{\kappa \cdot A} i$

## 1.2 magnetisches Feld

magnetische Feldstärke: $\vec{B} = \frac{\mu_0}{2\pi} \frac{i}{r} \cdot \vec{t}$

$\mu_0 = 4 \cdot \Pi \cdot 10^{-7}\ \text{N}/\text{A}^2$

magnetischer Fluß: $\Phi = \iint_A \vec{B} \cdot d\vec{A}$

$\oiint_A \vec{B} \cdot d\vec{A} = 0$

Kraft bei magnetischem Feld: $\vec{F} = q \cdot \left(\vec{v} \times \vec{B}\right)$

Kraft bei magnetischem und elektrischem Feld:

$$\vec{F} = q \cdot \left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Induktionsgesetz: $u_{12} = w \cdot \frac{d\Phi}{dt}$

Durchflutungsgesetz:

$$\oint_C \frac{\vec{B}}{\mu} \cdot dr = \Theta$$

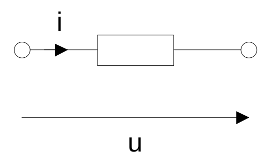
$$\Theta := \sum_{\nu} \left(\pm i_{\nu}\right)$$

# 2 Netzwerkelemente

## *2.1 Widerstand*

$u(t) = R \cdot i(t)$

$i(t) = \frac{u(t)}{R}$




$\underline{U} = R \cdot \underline{I}$

$\underline{I} = \frac{\underline{U}}{R}$

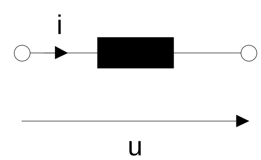
$[R] = 1\,{}^{V}\!/\!{}_{A} = 1\Omega$

$P = R \cdot I = \frac{U^2}{R}$

## *2.2 Induktivität*

$u(t) = L \cdot \frac{di(t)}{dt}$

$i(t) = i\left(t_0\right) + \frac{1}{L}\int_{t_0}^{t} u(\tau)d\tau$




$\underline{U} = j\omega L \cdot \underline{I}$

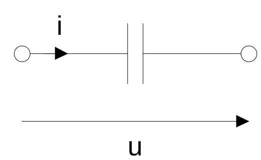
$\underline{I} = \frac{\underline{U}}{j\omega L}$

$[L] = 1\frac{Vs}{A} = 1H$

## *2.3 Kapazität*

$i(t) = C \cdot \frac{du(t)}{dt}$

$u(t) = u\left(t_0\right) + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$




$\underline{U} = \frac{\underline{I}}{j\omega C}$

$\underline{I} = j\omega C \cdot \underline{U}$

$[C] = 1\frac{As}{V} = 1F$

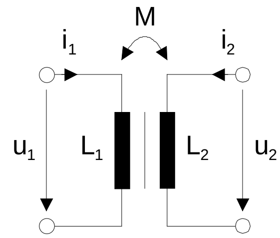
## *2.4 Übertrager*

### 2.4.1 realer Übertrager

$$u_1(t) = L_1 \cdot \frac{di_1(t)}{dt} + M \cdot \frac{di_2(t)}{dt}$$

$$u_2(t) = M \cdot \frac{di_1(t)}{dt} + L_2 \cdot \frac{di_2(t)}{dt}$$

$$L_1 > 0, L_2 > 0, M \neq 0, L_1 \cdot L_2 \geq M^2$$



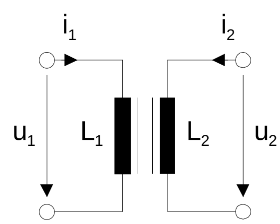

### 2.4.2 festgekoppelter Übertrager

(keine Streuflüsse)

$$M = \pm\sqrt{L_1 \cdot L_2}$$

(M<0, wenn Windungen unterschiedliche Richtungen haben)

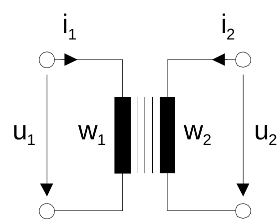
$$u_1 = \frac{w_1}{w_2} u_2$$




### 2.4.3 idealer Übertrager

(Permeabilität wächst über alle Grenzen)

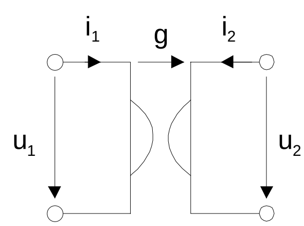
$$i_1 = -\frac{w_2}{w_1} i_2$$




## *2.5 Gyrator*

$$i_1(t) = g \cdot u_2(t) \qquad i_2(t) = -g \cdot u_1(t)$$

$$u_1(t) = -\frac{i_2(t)}{g} \qquad u_2(t) = \frac{i_1(t)}{g}$$




# 3 Komplexe Wechselstromrechnung

$$u(t) = \sqrt{2} \cdot U \cdot \cos(\omega \cdot t + \alpha) \Rightarrow \underline{U} = U \cdot e^{j\alpha}$$

$$i(t) = \sqrt{2} \cdot I \cdot \cos(\omega \cdot t + \beta) \Rightarrow \underline{I} = I \cdot e^{j\beta}$$

$$p(t) = 2 \cdot U \cdot I \cdot \cos(\omega \cdot t + \alpha) \cdot \cos(\omega \cdot t + \beta) \Rightarrow \underline{P} = \underline{U} \cdot \underline{I}^*$$

# 4 Zweitor-Matrizen

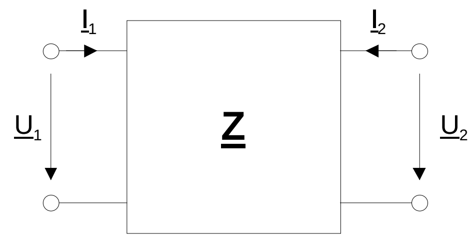
## *4.1 Übersicht über Zweitor-Matrizen*

### 4.1.1 Impedanzmatrix $\underline{Z}$

$$\begin{bmatrix}\underline{U}_1\\ \underline{U}_2\end{bmatrix}=\begin{bmatrix}\underline{z}_{11} & \underline{z}_{12}\\ \underline{z}_{21} & \underline{z}_{22}\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}\qquad \underline{\mathbf{Z}}=\begin{bmatrix}\underline{z}_{11} & \underline{z}_{12}\\ \underline{z}_{21} & \underline{z}_{22}\end{bmatrix}$$

$$\underline{z}_{11}=\left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{I_2=0}\qquad \underline{z}_{12}=\left.\frac{\underline{U}_1}{\underline{I}_2}\right|_{I_1=0}$$

$$\underline{z}_{21}=\left.\frac{\underline{U}_2}{\underline{I}_1}\right|_{I_2=0}\qquad \underline{z}_{22}=\left.\frac{\underline{U}_2}{\underline{I}_2}\right|_{I_1=0}$$

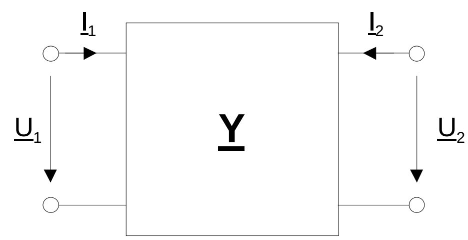



### 4.1.2 Admittanzmatrix $\underline{Y}$

$$\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}=\begin{bmatrix}\underline{y}_{11} & \underline{y}_{12}\\ \underline{y}_{21} & \underline{y}_{22}\end{bmatrix}\cdot\begin{bmatrix}\underline{U}_1\\ \underline{U}_2\end{bmatrix}\qquad \underline{\mathbf{Y}}=\begin{bmatrix}\underline{y}_{11} & \underline{y}_{12}\\ \underline{y}_{21} & \underline{y}_{22}\end{bmatrix}$$

$$\underline{y}_{11}=\left.\frac{\underline{I}_1}{\underline{U}_1}\right|_{U_2=0}\qquad \underline{y}_{12}=\left.\frac{\underline{I}_1}{\underline{U}_2}\right|_{U_1=0}$$

$$\underline{y}_{21}=\left.\frac{\underline{I}_2}{\underline{U}_1}\right|_{U_2=0}\qquad \underline{y}_{22}=\left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{U_1=0}$$

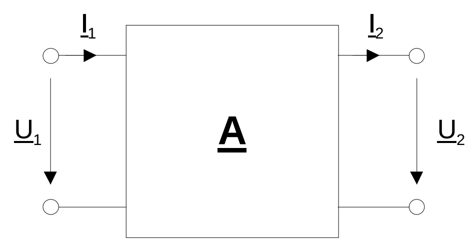



### 4.1.3 Kettenmatrix $\underline{A}$

$$\begin{bmatrix}\underline{U}_1\\ \underline{I}_1\end{bmatrix}=\begin{bmatrix}\underline{a}_{11} & \underline{a}_{12}\\ \underline{a}_{21} & \underline{a}_{22}\end{bmatrix}\cdot\begin{bmatrix}\underline{U}_2\\ \underline{I}_2\end{bmatrix}\qquad \underline{\mathbf{A}}=\begin{bmatrix}\underline{a}_{11} & \underline{a}_{12}\\ \underline{a}_{21} & \underline{a}_{22}\end{bmatrix}$$

$$\underline{a}_{11}=\left.\frac{\underline{U}_1}{\underline{U}_2}\right|_{I_2=0}\qquad \underline{a}_{12}=\left.\frac{\underline{U}_1}{\underline{I}_2}\right|_{U_2=0}$$

$$\underline{a}_{21}=\left.\frac{\underline{I}_1}{\underline{U}_2}\right|_{I_2=0}\qquad \underline{a}_{22}=\left.\frac{\underline{I}_1}{\underline{I}_2}\right|_{U_2=0}$$

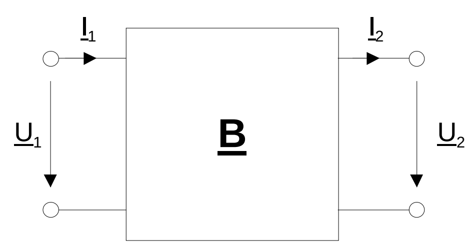



### 4.1.4 inverse Kettenmatrix $\underline{B}$

$$\begin{bmatrix}\underline{U}_2\\ \underline{I}_2\end{bmatrix}=\begin{bmatrix}\underline{b}_{11} & \underline{b}_{12}\\ \underline{b}_{21} & \underline{b}_{22}\end{bmatrix}\cdot\begin{bmatrix}\underline{U}_1\\ \underline{I}_1\end{bmatrix}\qquad \underline{\mathbf{B}}=\begin{bmatrix}\underline{b}_{11} & \underline{b}_{12}\\ \underline{b}_{21} & \underline{b}_{22}\end{bmatrix}$$

$$\underline{b}_{11}=\left.\frac{\underline{U}_2}{\underline{U}_1}\right|_{I_1=0}\qquad \underline{b}_{12}=\left.\frac{\underline{U}_2}{\underline{I}_1}\right|_{U_1=0}$$

$$\underline{b}_{21}=\left.\frac{\underline{I}_2}{\underline{U}_1}\right|_{I_1=0}\qquad \underline{b}_{22}=\left.\frac{\underline{I}_2}{\underline{I}_1}\right|_{U_1=0}$$

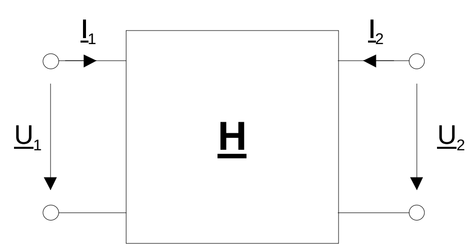



### 4.1.5 Hybridmatrix $\underline{H}$

$$\begin{bmatrix}\underline{U}_1\\ \underline{I}_2\end{bmatrix}=\begin{bmatrix}\underline{h}_{11} & \underline{h}_{12}\\ \underline{h}_{21} & \underline{h}_{22}\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_1\\ \underline{U}_2\end{bmatrix}\qquad \underline{\mathbf{H}}=\begin{bmatrix}\underline{h}_{11} & \underline{h}_{12}\\ \underline{h}_{21} & \underline{h}_{22}\end{bmatrix}$$

$$\underline{h}_{11}=\left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{U_2=0}\qquad \underline{h}_{12}=\left.\frac{\underline{U}_1}{\underline{U}_2}\right|_{I_1=0}$$

$$\underline{h}_{21}=\left.\frac{\underline{I}_2}{\underline{I}_1}\right|_{U_2=0}\qquad \underline{h}_{22}=\left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{I_1=0}$$



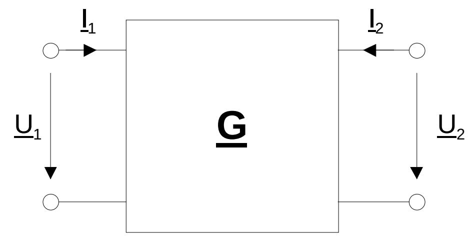
### 4.1.6 inverse Hybridmatrix $\underline{G}$

$$\begin{bmatrix}\underline{I}_1\\ \underline{U}_2\end{bmatrix}=\begin{bmatrix}\underline{g}_{11} & \underline{g}_{12}\\ \underline{g}_{21} & \underline{g}_{22}\end{bmatrix}\cdot\begin{bmatrix}\underline{U}_1\\ \underline{I}_2\end{bmatrix}$$

$$\underline{\mathbf{G}}=\begin{bmatrix}\underline{g}_{11} & \underline{g}_{12}\\ \underline{g}_{21} & \underline{g}_{22}\end{bmatrix}$$

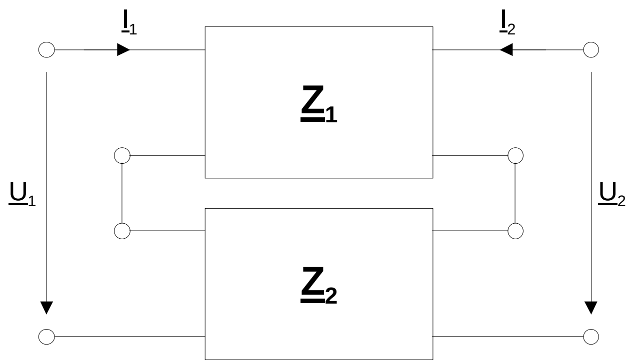
$$\underline{g}_{11}=\frac{\underline{I}_1}{\underline{U}_1}\bigg|_{I_2=0}\qquad \underline{g}_{12}=\frac{\underline{I}_1}{\underline{I}_2}\bigg|_{U_1=0}$$

$$\underline{g}_{21}=\frac{\underline{U}_2}{\underline{U}_1}\bigg|_{I_2=0}\qquad \underline{g}_{22}=\frac{\underline{U}_2}{\underline{I}_2}\bigg|_{U_1=0}$$



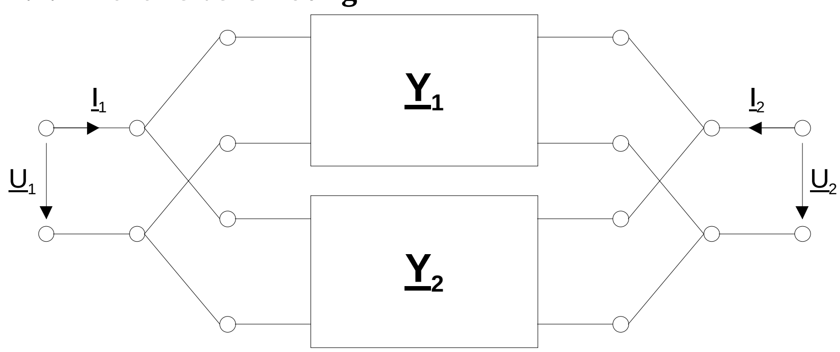

## *4.2 Zweitorverbindungen*

### 4.2.1 Reihenverbindung



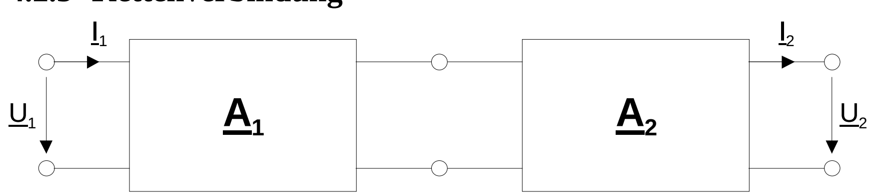

$$\underline{\mathbf{Z}}_{\mathbf{ges}}=\underline{\mathbf{Z}}_{\mathbf{1}}+\underline{\mathbf{Z}}_{\mathbf{2}}$$

### 4.2.2 Parallelverbindung




$$\underline{\mathbf{Y}}_{\mathbf{ges}}=\underline{\mathbf{Y}}_{\mathbf{1}}+\underline{\mathbf{Y}}_{\mathbf{2}}$$

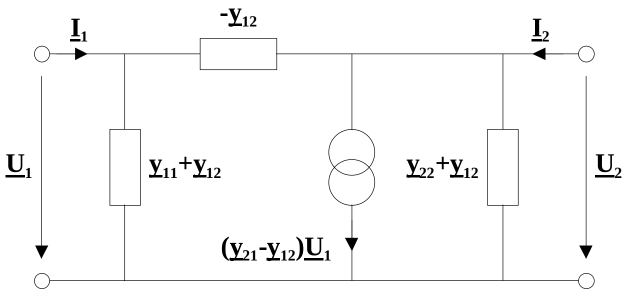
### 4.2.3 Kettenverbindung



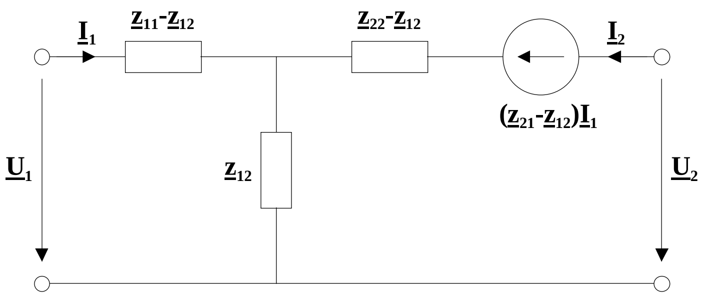

$$\underline{\mathbf{A}}_{\mathbf{ges}}=\underline{\mathbf{A}}_{\mathbf{1}}\cdot\underline{\mathbf{A}}_{\mathbf{2}}$$

## *4.3 Ersatzschaltbilder für Matrizen*

### 4.3.1 π-Ersatzschaltbild

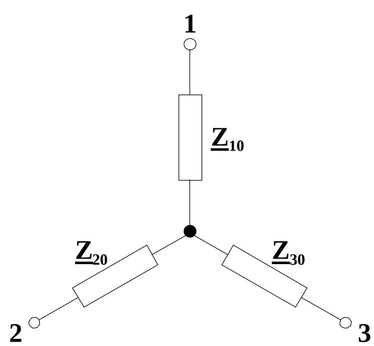


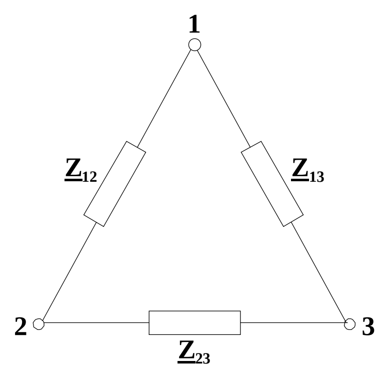

### 4.3.2 T-Ersatzschaltbild




# 5 Vermischtes

## *5.1 Stern-Dreieck-Transformation*




$$\underline{Z}_{10} = \frac{\underline{Z}_{12} \cdot \underline{Z}_{13}}{\underline{Z}_{12} + \underline{Z}_{13} + \underline{Z}_{23}}$$

$$\underline{Z}_{20} = \frac{\underline{Z}_{12} \cdot \underline{Z}_{23}}{\underline{Z}_{12} + \underline{Z}_{13} + \underline{Z}_{23}}$$

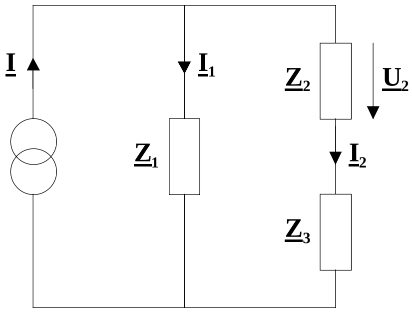
$$\underline{Z}_{30} = \frac{\underline{Z}_{13} \cdot \underline{Z}_{23}}{\underline{Z}_{12} + \underline{Z}_{13} + \underline{Z}_{23}}$$

$$\underline{Z}_{12} = \frac{\underline{Z}_{10} \cdot \underline{Z}_{20} + \underline{Z}_{20} \cdot \underline{Z}_{30} + \underline{Z}_{30} \cdot \underline{Z}_{10}}{\underline{Z}_{30}}$$

$$\underline{Z}_{23} = \frac{\underline{Z}_{10} \cdot \underline{Z}_{20} + \underline{Z}_{20} \cdot \underline{Z}_{30} + \underline{Z}_{30} \cdot \underline{Z}_{10}}{\underline{Z}_{10}}$$

$$\underline{Z}_{13} = \frac{\underline{Z}_{10} \cdot \underline{Z}_{20} + \underline{Z}_{20} \cdot \underline{Z}_{30} + \underline{Z}_{30} \cdot \underline{Z}_{10}}{\underline{Z}_{20}}$$

### 5.2 erweiterte Spannungsteilerschaltung



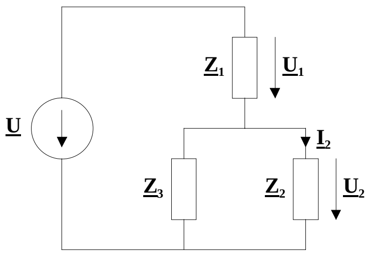

$$\underline{Z}_{ges} = \frac{\underline{Z}_1 \cdot \left(\underline{Z}_2 + \underline{Z}_3\right)}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3} \qquad \underline{Y}_{ges} = \frac{\underline{Y}_1 \cdot \underline{Y}_2 + \underline{Y}_1 \cdot \underline{Y}_3 + \underline{Y}_2 \cdot \underline{Y}_3}{\underline{Y}_2 + \underline{Y}_3}$$

$$\underline{I}_1 = \frac{\underline{Z}_2 + \underline{Z}_3}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3} \cdot \underline{I} \qquad \underline{I}_1 = \frac{\underline{Y}_1 \cdot \left(\underline{Y}_2 + \underline{Y}_3\right)}{\underline{Y}_1 \cdot \underline{Y}_2 + \underline{Y}_1 \cdot \underline{Y}_3 + \underline{Y}_2 \cdot \underline{Y}_3} \cdot \underline{I}$$

$$\underline{I}_2 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3} \cdot \underline{I} \qquad \underline{I}_2 = \frac{\underline{Y}_2 \cdot \underline{Y}_3}{\underline{Y}_1 \cdot \underline{Y}_2 + \underline{Y}_1 \cdot \underline{Y}_3 + \underline{Y}_2 \cdot \underline{Y}_3} \cdot \underline{I}$$

$$\underline{U}_2 = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3} \cdot \underline{I} \qquad \underline{U}_2 = \frac{\underline{Y}_3}{\underline{Y}_1 \cdot \underline{Y}_2 + \underline{Y}_1 \cdot \underline{Y}_3 + \underline{Y}_2 \cdot \underline{Y}_3} \cdot \underline{I}$$

### 5.3 erweiterte Stromteilerschaltung




$$\underline{Z}_{ges} = \frac{\underline{Z}_1 \cdot \underline{Z}_2 + \underline{Z}_1 \cdot \underline{Z}_3 + \underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} \qquad \underline{Y}_{ges} = \frac{\underline{Y}_1 \cdot \left(\underline{Y}_2 + \underline{Y}_3\right)}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3}$$

$$\underline{U}_1 = \frac{\underline{Z}_1 \cdot \left(\underline{Z}_2 + \underline{Z}_3\right)}{\underline{Z}_1 \cdot \underline{Z}_2 + \underline{Z}_1 \cdot \underline{Z}_3 + \underline{Z}_2 \cdot \underline{Z}_3} \cdot \underline{U} \qquad \underline{U}_1 = \frac{\underline{Y}_2 + \underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3} \cdot \underline{U}$$

$$\underline{U}_2 = \frac{\underline{Z}_2 \cdot \underline{Z}_3}{\underline{Z}_1 \cdot \underline{Z}_2 + \underline{Z}_1 \cdot \underline{Z}_3 + \underline{Z}_2 \cdot \underline{Z}_3} \cdot \underline{U} \qquad \underline{U}_2 = \frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3} \cdot \underline{U}$$

$$\underline{I}_2 = \frac{\underline{Z}_3}{\underline{Z}_1 \cdot \underline{Z}_2 + \underline{Z}_1 \cdot \underline{Z}_3 + \underline{Z}_2 \cdot \underline{Z}_3} \cdot \underline{U} \qquad \underline{I}_2 = \frac{\underline{Y}_1 \cdot \underline{Y}_2}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3} \cdot \underline{U}$$

### 5.4 Schwingkreis

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}$$

$$Q = \begin{cases} \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 RC} = \dfrac{1}{R} \cdot \sqrt{\dfrac{L}{C}} := Q_R & \text{im Reihenschwingkreis} \\ \dfrac{R}{\omega_0 L} = \omega_0 RC = R \cdot \sqrt{\dfrac{C}{L}} := Q_P & \text{im Parallelschwingkreis} \end{cases} \qquad \text{wobei } Q_R = \frac{1}{Q_P}$$

# 6 Mathematischer Anhang

## 6.1 Lineare Gleichungssysteme

### 6.1.1 Berechnung von Determinanten

**1. Determinante zweiter Ordnung**

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

**2. Determinante dritter Ordnung**

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

**3. Determinante n-ter Ordnung**

Beispiel: Entwicklung nach der 3-ten Zeile

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = a_{31} \cdot \begin{vmatrix} a_{12} & a_{13} & a_{14} \\ a_{22} & a_{23} & a_{24} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} - a_{32} \cdot \begin{vmatrix} a_{11} & a_{13} & a_{14} \\ a_{21} & a_{23} & a_{24} \\ a_{41} & a_{43} & a_{44} \end{vmatrix} + a_{33} \cdot \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} - a_{34} \cdot \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{41} & a_{42} & a_{43} \end{vmatrix}$$

Die Vorzeichen ergeben sich gemäß des Tableaus.

$$\begin{matrix} + & - & + & \cdots \\ - & + & - & \cdots \\ + & - & + & \cdots \\ \vdots & \vdots & \vdots & \end{matrix}$$

### 6.1.2 Die CRAMER'sche Regel

Die Lösungen $x_n$ eines linearen Gleichungssystems können auf folgende Weise bestimmt werden.

$$x_k = \frac{\Delta_k}{\Delta} \qquad k = 1 \ldots n$$

$\Delta$ ist die Determinante des Gleichungssystems; bei der Determinante $\Delta_k$ wurde die k-te Spalte durch die rechte Seite ersetzt wurde.

## *6.2 Komplexe Zahlen*

### 6.2.1 Betrag

$$\underline{Z} := \frac{a + jb}{c + jd} \Rightarrow |\underline{Z}| = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$$

### 6.2.2 EULER'sche Formel

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$e^{-j\varphi} = \cos\varphi - j\sin\varphi$$

$$\cos\varphi = \tfrac{1}{2}\left(e^{j\varphi} + e^{-j\varphi}\right) = \operatorname{Re}\left(e^{j\varphi}\right)$$

$$\sin\varphi = \tfrac{1}{2j}\left(e^{j\varphi} - e^{-j\varphi}\right) = \operatorname{Im}\left(e^{j\varphi}\right)$$

### 6.2.3 Argument

$$\underline{Z} := a + jb \Rightarrow \arg(Z) = \begin{cases} \arctan\left(\frac{b}{a}\right) & \text{für } a > 0,\ b \neq 0 \\ \arctan\left(\frac{b}{a}\right) + \Pi & \text{für } a < 0,\ b > 0 \\ \arctan\left(\frac{b}{a}\right) - \Pi & \text{für } a < 0,\ b < 0 \end{cases}$$

$$\arg\left(\frac{\underline{Z}}{\underline{N}}\right) = \arg(\underline{Z}) - \arg(\underline{N})$$

### 6.2.4 Wurzeln

$$\underline{Z} = z \cdot e^{j\varphi} \Rightarrow \sqrt[n]{\underline{Z}} = \sqrt[n]{z} \cdot e^{j\frac{\varphi + 2k\Pi}{n}}, k = 0, \ldots, n-1$$

## *6.3 Matrix-Multiplikation*

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} \end{bmatrix}$$

## *6.4 Trigonometrische Funktionen*

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha$$

$$\sin^2 \alpha = \tfrac{1}{2}(1 - \cos 2\alpha)$$

$$\cos^2 \alpha = \tfrac{1}{2}(1 + \cos 2\alpha)$$

$$\cos ax \cdot \sin bx = \tfrac{1}{2}\left(\sin(a+b)x - \sin(a-b)x\right)$$

$$\cos ax \cdot \cos bx = \tfrac{1}{2}\left(\cos(a-b)x + \cos(a+b)x\right)$$

## *6.5 Geometrie*

### 6.5.1 Kreis

Umfang: $u = 2 \cdot r \cdot \pi$
Flächeninhalt: $A = r^2 \cdot \pi$

### 6.5.2 Kugel

Oberfläche: $O = 4 \cdot r^2 \cdot \pi$
Volumen: $V = \tfrac{4}{3} \cdot r^3 \cdot \pi$